A Software-Based Network Analyser

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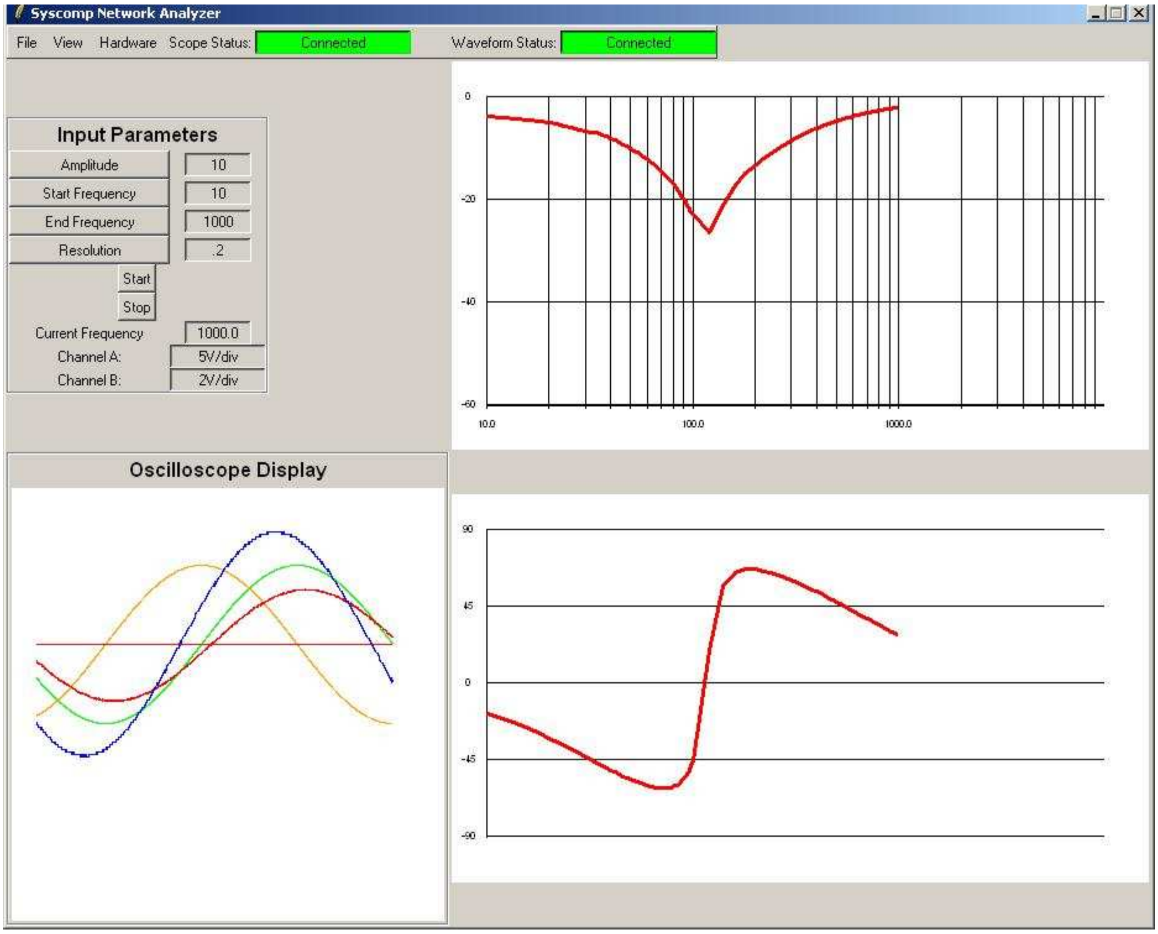

1 Overview

The purpose of a *network analyser* is to determine the gain and phase response or *transfer function* of some network. The GUI (graphical user interface) of the network analyser described in this paper, is shown on the cover page.

The plot in the upper right corner is the amplitude response of a 120Hz notch filter. The plot in the lower right corner is the phase response of the same filter.

In this paper, we describe a program that operates the Syscomp WGM-101 Waveform Generator and DSO-101 Oscilloscope to do network analysis over the frequency range 0.1 Hz to 100kHz.

The process is illustrated in figure 1 on page 3. It may be useful to refer to this diagram during the development of the equations, below.

The network is driven by a sine wave at some frequency ω. The oscilloscope captures the input signal to the network on Channel A and the output signal on Channel B. The network analyser software then determines the relative amplitude between input and output of the network to get the magnitude of the transfer function, often referred to as the *gain* or *loss* of the network. The network analyser software also compares the phases of the input and output sine waves to determine the phase shift through the network.

The network analyser software plots the magnitude and phase of the transfer function over the frequency range of interest. This is a very useful characterization of a network.

Although this sounds straightforward, it is complicated by the presence of any noise on either of these signals. The strategy given below is immune to noise and allows magnitude and phase measurement over a larger range than simple techniques.

1.1 Measurement Strategy

The network analyser software is operating the waveform generator hardware, so it knows the frequency setting of the generator at all times. Therefore it is possible to create in software a sine wave at that frequency. This sine wave e_{int} can be use to extract amplitude and phase from both the network reference signal e_{ref} and the network output signal e_{out}. This process involves integration, so it is inherently noise free. (This is the technique used in the *lock-in amplifier*, a very common method of physics measurement.)

Once the amplitude and phase of e_{ref} and e_{out} are known, then the transfer function of the network is the simply ratio of the amplitudes and the difference of the phases.

2 Derivation

Consider that the generator is being operated at some radian frequency ω radians/second. In software, it is then possible to generate the functions $\sin\omega t$ and $\cos\omega t$. These will be taken as the *internal sine waves* which have an amplitude of unity and a phase of zero degrees – that is, they are the reference for phase measurements.

2.1 In-phase component of the reference

The input signal to the network may be represented by:

$$e_{ref} = E_{ref}\sin(\omega t + \alpha) \tag{1}$$

Both the peak amplitude E_{ref} and the phase angle α are unknown.

Now suppose that we capture e_{ref} with the oscilloscope and then multiply it by $\sin\omega t$. Use the trig identity

$$\sin A\ \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right) \tag{2}$$

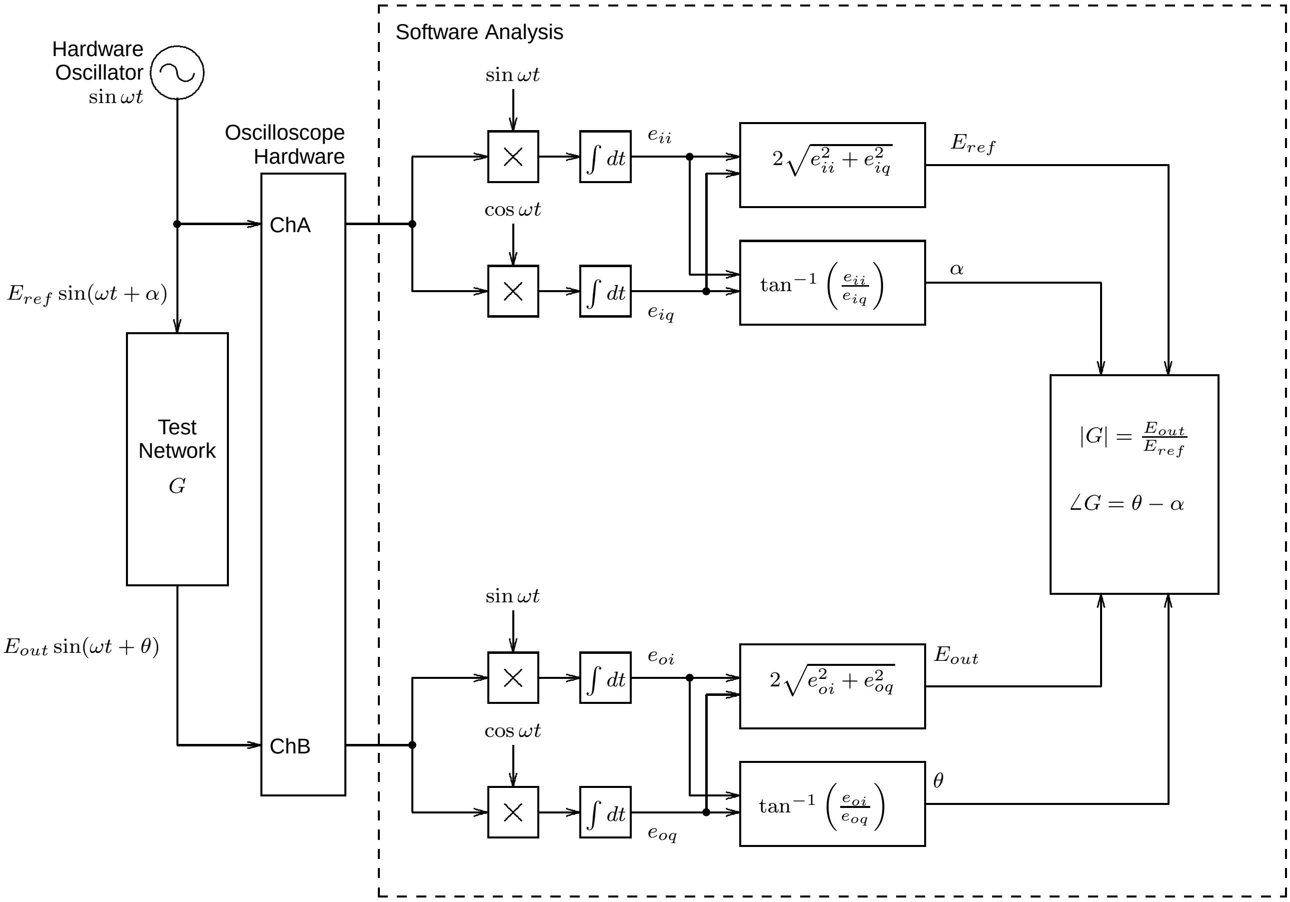

Figure 1: Network Analyser, Block Diagram

Substitute $A=\sin\omega t$ and $B=E_{ref}\sin(\omega t+\alpha)$, and the result of this (call it e_A) is:

$$e_A=\frac{E_{ref}}{2}\left(\cos\alpha-\cos(2\omega t+\alpha)\right) \tag{3}$$

In frequency domain terms, the multiplication process generated the difference frequency and sum frequency. The difference frequency is a DC value of

$$e_{Adc}=\frac{E_{ref}}{2}\cos\alpha \tag{4}$$

The sum frequency is

$$e_{Aac}=\frac{E_{ref}}{2}\cos(2\omega t+\alpha) \tag{5}$$

that is, an AC waveform at twice the signal frequency.

If we now *integrate* the output signal of the multiplier e_A, over one or more complete cycles of the waveform, the AC component will integrate to zero, leaving the DC component e_{Adc}.

This is known as the *in-phase* component of the reference voltage, so we'll write it as:

$$e_{ii} = \frac{E_{ref}}{2} \cos \alpha \tag{6}$$

2.2 Quadrature component of the reference

Now repeat this process: multiply the digitized reference signal by the cosine version of the internal sine wave, $\cos \omega t$.

The result of the multiplication and integration is

$$e_{iq} = \frac{E_{ref}}{2} \sin \alpha \tag{7}$$

2.3 Determining the Amplitude and Phase of the Reference Signal

Equations 6 and 7 represent vectors that are two sides of a right angle triangle, where $E_{ref}/2$ is the magnitude of the hypotenuse. Consequently, we can solve for the amplitude of E_{ref} with

$$E_{ref} = 2\sqrt{e_{ii}^2 + e_{iq}^2} \tag{8}$$

The phase angle α is given by:

$$\alpha = \tan^{-1}\left(\frac{e_{ii}}{e_{iq}}\right) \tag{9}$$

The value of α is the angle of the *reference* signal, at the input to the network, with respect to the internal software constructed sine wave. In practice, this phase angle will vary with the trigger level of the oscilloscope. For example, it would be close to zero degrees if the scope trigger level is at the zero crossing of the positive slope of the signal.

2.4 Determining the Amplitude and Phase of the Output Signal

We can treat the output signal in exactly the same fashion of the reference signal.

Suppose the output from the network signal can be represented as:

$$e_{out} = E_{out} \sin \omega t + \theta \tag{10}$$

where θ is the phase angle of the output signal with respect to the internal sine wave.

Multiplying e_{out} by $\sin \omega t$ and $\cos \omega t$ and then integrate the result. Apply the hypotenuese equation and obtain the magnitude E_{out}, then, as α was obtained previously, solve for the phase angle θ.

2.5 Final Steps

Our reference and output signals are now determined. In phasor notation:

$$\begin{aligned} e_{ref} &= E_{ref}\angle\alpha \\ e_{out} &= E_{out}\angle\theta \end{aligned}$$

The transfer function of the network, magnitude and phase, at this particular frequency is then:

$$G(\omega) = \frac{E_{out}}{E_{ref}} \angle(\theta - \alpha) \tag{11}$$

3 Implementation Notes

3.1 Gain Ranging

Although the correlation technique is very noise immune, it must have some signal to work with. The A/D converter in the oscilloscope is an 8 bit device, so its dynamic range is limited to something in the order of 48db. However, the oscillosope has a variable gain preamplifier with seven different gain settings, so this is adjusted when the signal falls above maximum or below minimum threshold.

3.2 Removal of DC Component

It is best to remove any DC component that may exist in a waveform before it is applied to the input of a multiplier in figure 1. A DC component will cause the software-generated sine wave to leak into the multiplier output. If it is small, this waveform will be removed without effect during the integration stage. So, strictly speaking, this is not essential. However, it simplifies analysis and debugging.

This is accomplished by determining the average value of the input waveform and subtracting it from the signal.

3.3 Integration

In a noiseless system, it is sufficient to integrate over one cycle of the waveform. However, when noise is present, it is best to integrate over the longest possible record of complete AC cycles. Any AC component will then integrate to zero. As the record length increases, the DC component (which is the measurement signal we are after), will accumulate while the noise component (which is undesired) is more likely to integrate to zero. Consequently, the system uses as much of the oscilloscope waveform memory as possible.

4 Further Reading

A search with Google for ’lock-in amplifier’ will turn up material that explains other examples this technique. The process of multiplying a signal by a sine wave, and then integrating (or low-pass filtering) the output of the multiplier, is known as *correlation*. In effect, the circuit determines the similarity between the two inputs to the multiplier. Correlation is extermely useful and appears in many forms in communications and instrumentation circuits.