Measuring AC Current and Power Factor

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Introduction

The power factor of an AC installation describes the phase relationship of the voltage and current. If the load is reactive, then the current is out of phase with the voltage. In the context of an AC power distribution, a reactive load puts the power company at an economic disadvantage: they can only bill for the power supplied (that is, the voltage and current that are in phase) but they actually have to supply a larger current, and they need to provide the infrastructure to deliver that current. For that reason, power companies may penalize customers with a non-resistive load [1].

When the voltage and current are sine waves, then the power factor is related to their phase relationship. The phase relationship can be measured by various specialized instruments, or an oscilloscope.

For some loads, the current waveform is a complex shape, and then an oscilloscope is essential for measuring the shape of the current waveform. If the oscilloscope has the capability of spectrum analysis, then one can measure and calculate the harmonic content of the current. The power factor is then related to the magnitude of the fundamental component and the harmonics.

In these notes, we have specifically in mind the CGR-101 oscilloscope [2], but the information is applicable to anyone measuring power factor.

Power Factor for Sinusoidal Current

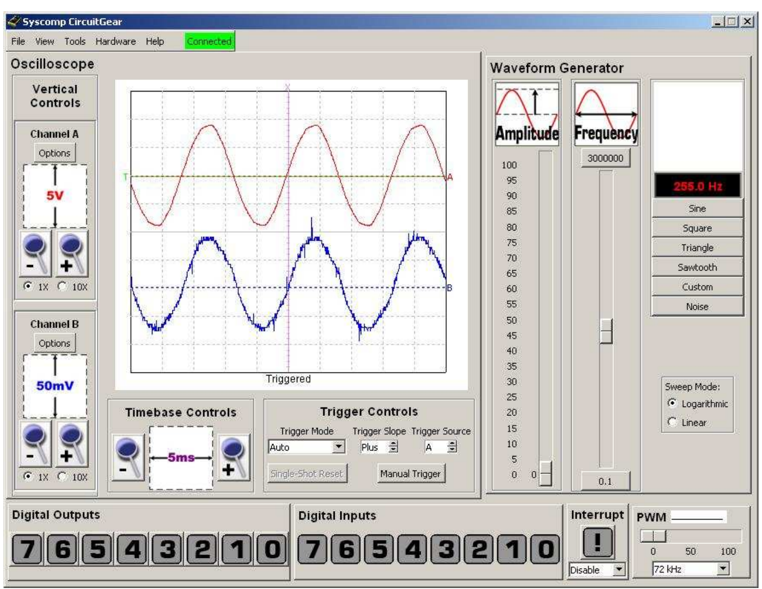


Figure 1: 60 Watt Lamp Load. The red trace is proportional to the AC line voltage. The blue trace is proportional to the current waveform. They are in phase because the load is resistive.

When the load is resistive, the AC voltage and current are in phase and the power delivered to the load is the product of the two, in watts [1]. Figure 1 shows an example of the voltage and current phase relationship for a resistive load.

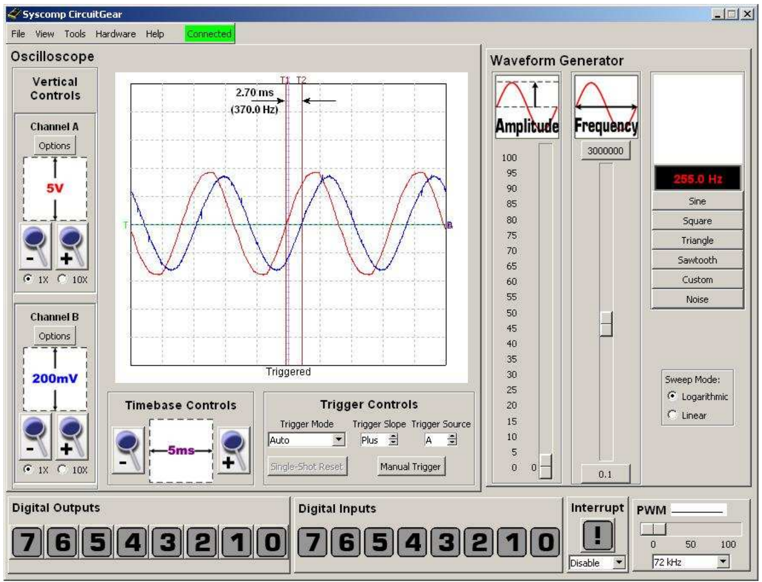


Figure 2: Electric Fan Load. The red trace is proportional to the AC line voltage. The blue trace is proportional to the current waveform. The current lags the voltage because the load is somewhat inductive. The time measurement cursors help determine the magnitude of the lagging phase angle.

When the load has some reactive component, then the current and voltage are no longer in phase. For example, if the load is an electric motor, then the coils of wire in the motor cause the current to lag the voltage. Figure 2 shows an example.

In that case, it is useful to treat the current I as being composed of the sum of two quadrature vectors: the *in-phase* component I_p, which is in phase with the voltage, and the *quadrature* component I_q, which is 90° out of phase with the voltage. The vector sum of I_p and I_q is the original current I.

In terms of the phase angle ϕ between voltage and current, the in-phase component of current is given by:

$$I_p = I\cos(\phi) \tag{1}$$

$$I_q = I\sin(\phi) \tag{2}$$

The product of the *in-phase* component of current and the voltage is the *power* P (sometimes called the *resistive* power) in the load. This is the power that is doing useful work, measured in watts.

The product of the *quadrature* component of current and the voltage is the *reactive power* Q, measured in VAR (*volt-amps reactive*). It does no useful work. In effect, it is power that is borrowed from the power distribution company and then returned to it, over one AC cycle.

The power factor PF is equal to the cosine of the phase angle:

$$PF = \cos(\phi) \tag{3}$$

$$S = \sqrt{P^2 + Q^2} \tag{4}$$

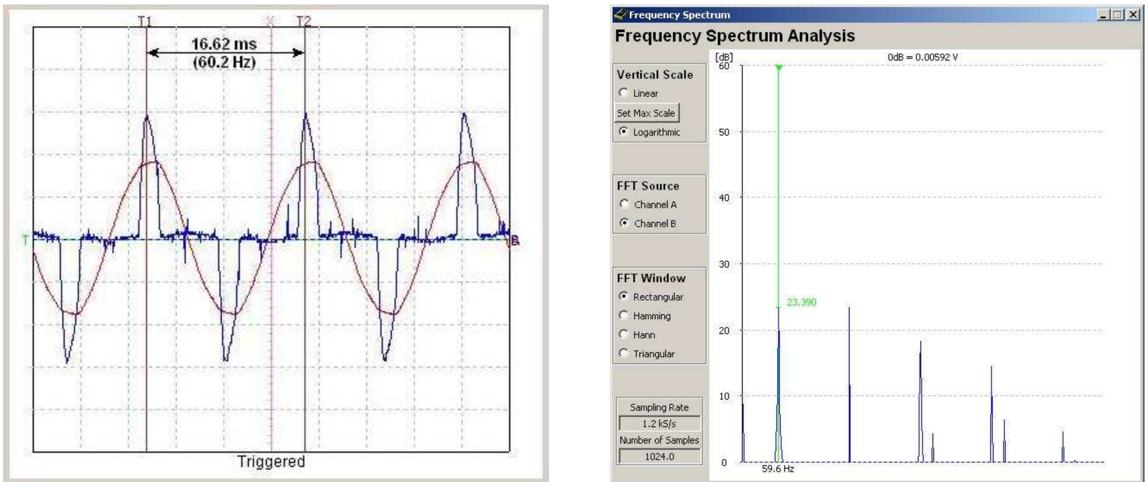
In some jurisdictions, the distribution company bills for S, the vector sum of the resistive and reactive power.

Reducing the reactive power Q will reduce the power bill so it's of great interest to industries to *correct the power factor* as the process is called. For minimum reactive power, and to minimize the cost of electricity, the

[1] The *voltage* and *current* described here are the *effective* or *RMS* values. This becomes relevant when we do measurements with an oscilloscope, where the effective value of a waveform is equal to $1/\sqrt{2} = 0.707$ times the peak value.

power factor should be close to unity, that is, the voltage and current should be in phase. For example, if the load is inductive, the line can be shunted by capacitors until the power factor is close to unity. Reference [4] describes this process in detail.

Power Factor for Non-Sinusoidal Current



(a) Waveforms (Red, Voltage: Blue, Current

(b) Spectrum (Log scale vertical) of the Current Waveform

Figure 3: Full Wave Bridge Rectifier, Voltage and Current Waveforms

Many electronic loads create a non-sinusoidal line current. For example, consider a line-operated power supply consisting of a full-wave bridge rectifier with capacitor filter, similar to the schematic in figure 4. The current, blue trace in figure 3(a), consists of two short pulses per cycle of the AC voltage. The peak value of these pulses is *much* larger than the RMS value, again representing a problem for the power distribution company. Their infrastructure must provide for these peak currents, even though they bill at the much lower value of the effective (RMS) current.

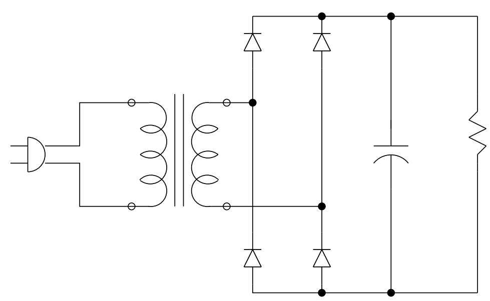


Figure 4: Full-Wave Bridge Rectifier and Capacitor Filter

It's useful to consider the current waveform in terms of its Fourier series: it consists of a fundamental component (at 60Hz in North America, for example) plus harmonic components at multiples of the fundamental, as shown in the CGR-101 spectrum analysis display of figure 3(b). A pure sine wave would show up as one vertical trace in the spectrum. This complex waveform shows a number of traces: one at the fundamental (60Hz) and additional traces at odd harmonics of the fundamental.

The *distortion power factor* D is defined as:

$$D = \frac{I_1}{I_T} \tag{5}$$

where I_1 is the rms value of the fundamental component of current and I_T is the rms value of the total current.

Let us assume that the waveform consists of a fundamental component, at the AC line frequency, of magnitude I_1. There is also a second harmonic I_2 and a third harmonic of I_3. (There are usually higher order harmonics, but let's assume they are negligible in amplitude and can be neglected.) Then the total current is given by:

$$I_T{}^2 = I_1{}^2 + I_2{}^2 + I_3{}^2 \quad (6)$$

Let us assume that the voltage waveform is a pure sine wave. (This is usually a valid assumption.) Power is only generated when the voltage and current are of the same frequency. Consequently, in equation 6, only the fundament component I_1 generates real power. The other components (I_2, I_3) contribute to the RMS current – and worsen the power factor – but do not generate real power. If we do a spectrum analysis of the current waveform, that will give us the relative magnitude of the fundamental component I_1 and the harmonics, I_2 and I_3. For example, for a square waveform of current, the second harmonic is zero and the third harmonic is one-third of the amplitude of the fundamental. Then we can write:

$$I_2 = K_2 I_1, \quad I_3 = K_3 I_1$$

The K values are typically – but not always – less than unity. Back-substitute these values into equation 6:

$$\begin{aligned} I_T{}^2 &= I_1{}^2 + I_2{}^2 + I_3{}^2 \\ &= I_1{}^2 + K_2 I_1{}^2 + K_3 I_1{}^2 \\ &= I_1{}^2 \left[1 + K_2{}^2 + K_3{}^2\right] \end{aligned} \quad (7)$$

Equation 6 is useful because it relates the magnitude of the fundamental component I_1 to the spectrum components K_2 and K_3 and to I_T, the total value of the RMS current. Both the spectrum and the total RMS current can be measured, so we have a way of calculating the fundamental component of current I_1. Then we can substitute the total and fundamental current values into equation 5 to determine the distortion power factor D.

This process would enable us to determine if the power supply meets a specified requirement for distortion power factor.

Measuring Power Factor

When measuring line voltages and currents with an oscilloscope, you need to be very, very careful about connections. Connecting the common on the oscilloscope to the live connection on the AC line will immediately destroy the instrument. Connecting the common to the neutral is a bit safer, but the current on the neutral connection now has a parallel path back to the source, and that can cause problems or damage.

The simplest way to measure voltage - safely - is to use a transformer to isolate and step down the voltage waveform. A readily available step-down power transformer would be one possible choice. The primary is connected to the line. The oscilloscope is connected to the secondary. The operator then scales the voltage waveform by the stepdown ratio of the transformer.

The current waveform can be measured with a clip-on ammeter (clamp meter) which has a waveform output connection. This is perfectly safe, because the clip-on ammeter has no physical connection to the AC line. The IES ES-695 [6] is one possible such meter.

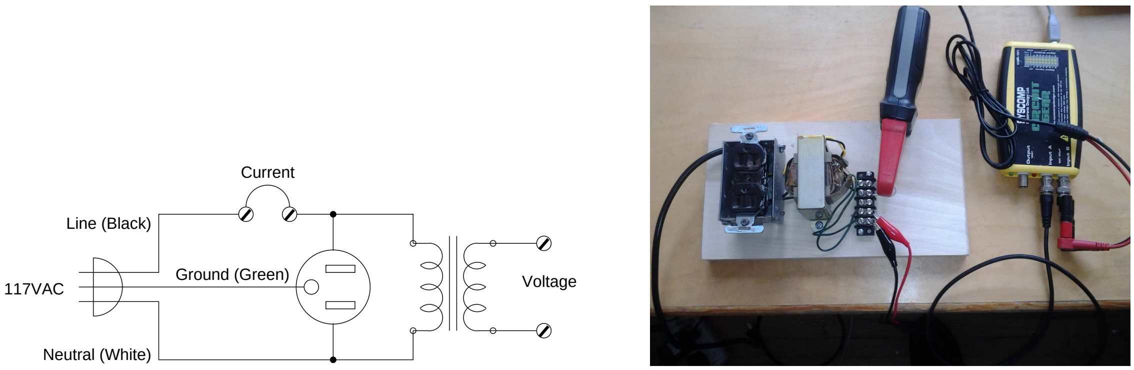
An alternative to the clip-on ammeter is the split core transformer [7] available from Sparkfun Electronics [8]. This device is much less expensive. It can only measure the AC component in the waveform, but that's acceptable in most AC current monitoring applications.

Breakout Adaptor for Power Factor Measurements

An adaptor board was constructed to make some power factor measurements – see figure 5.

Figure 5(a) shows the schematic. A line cord brings AC power to a receptacle, where various loads can be attached. The AC line is brought to two *current* terminals. A clamp-meter can clip onto a jumper between these two terminals. The transformer is used to isolate and step down the line voltage. In this case, the transformer is nominally 117VAC:6.3VAC, but anything similar will do. The current rating is unimportant.

In the schematic of figure 5(a), the measured current includes the transformer current. Since the transformer is open-circuited for a measurement of line voltage, the current in the transformer primary is the *magnetizing current*, which is small enough to be ignored when compared to most real-world load currents. This arrangement



(a) Schematic

(b) Construction

Figure 5: Adaptor Board for Power Factor Measurement

allows one to connect a low voltage load to the transformer secondary and then measure the current in the primary. Alternatively, the transformer connection could be moved to before the current jumper, so that the transformer current does not add to the measured current.

Figure 5(b) shows the construction. The IES ES-695 clip-on ammeter is shown attached to the jumper between the current terminals. The output from the ammeter goes to channel B of the CGR-101 oscilloscope[2]. The voltage terminals are connected to channel A of the CGR-101 oscilloscope.

If you decide to build a breakout panel for these measurements, it is strongly recommended that you provide proper safety shielding and connections. The construction in figure 5(b) is unsafe for the following reasons:

- **The current terminals have an unshielded, potentially lethal connection to the AC line.**
- **Connecting the oscilloscope inputs to the current terminals, by mistake, will instantly and completely destroy the oscilloscope.**

If students are doing these measurements, then it would be adviseable to provide a lower-voltage system, or one in which it is not possible to come in direct contact with the 117VAC line potential.

Calibration

The adaptor board needs to be calibrated. To determine the line voltage, we must know the turns ratio of the transformer. Measure the primary and secondary voltages with an AC voltmeter, and their ratio is the turns ratio, which we will call N_v. In our case we measured

$$N_v = \frac{V_{pri}}{V_{sec}} = \frac{113}{6.7} = 6.8 \text{ volts per volt}$$

We also need to know the calibration factor of the current meter. According to the IES-695 manual, on the 20A scale the current meter produces 100mV per ampere of current. On the current meter 80A scale it's 10mV per amp.

$$N_i = 0.1 \text{ volts/amp}$$

Now we are in a position to make measurements.

Example: Power Factor (Sinusoidal Waveforms)

Consider first the electric fan load of figure 2. The voltage and current are both sinusoidal waveforms, so the calculation of power factor is given by equation 3. Consequently, we need to find the phase angle between the two waveforms.

[2]The plugs on the current probe cable are protected by plastic shields. It's necessary to cut those away to plug them into a BNC-Binding post adaptor for the oscilloscope. This is safe to do because there are no high voltages on these terminals. It's also necessary to spread the plug sections slightly to get good contact in the adaptor.

The screen shot of figure 2 shows time cursors measuring the time between zero crossings of the current and voltage waveforms. At the zero crossing, the current lags the voltage by 2.7mSec. A full AC cycle is 1/60=16.6 mSec. Then 2.7 msec corresponds to a phase angle of:

$$\phi = 360 \times \frac{2.7}{16.6} = 58 \text{ degrees}$$

Then the power factor, according to equation 1, is:

$$PF = \cos(\phi) = 0.52$$

Example: Distortion Power Factor (Non-Sinusoidal Waveforms

Now we will determine the distortion power factor for the waveform shown in figure 3(a).

The harmonics of the current waveform we determined from the CGR-101 spectrum analysis display, shown in figure 3(b). The harmonics are listed in the following table[3]:

Harmonic	Frequency, Hz	Magnitude, db_v	K_n Name	K_n Magnitude
Fundamental	60	23.4	K_1	1
Third	180	23	K_3	0.95
Fifth	300	18.5	K_5	0.57
Seventh	420	14	K_7	0.34
Ninth	540	5	K_9	0.12

Notice that magnitude of the the even harmonics (2nd, 4th, 6th . . .) is zero.

Here's how we determined the harmonic coefficients listed in the last column of the table:

The magnitude in db_v for the fundamental (let's call it Mag_1db) is given by

$$Mag_1db = 20 \log_{10} \frac{V}{V_{ref}}$$

(V_{ref} is some reference voltage, which need not concern us, since it will soon be cancelled and consigned to outer darkness.) Running this backward to find V_1, we have:

$$V_1 = V_{ref} 10^{\left[\frac{Mag_1db}{20}\right]} \tag{8}$$

A similar relationship applies for V_2 and the other harmonics. The coefficient listed in the last column of the table is ratio of $V_{whatever}$ to V_1. For example:.

$$\begin{aligned} K_3 &= \frac{V_3}{V_1} \\ &= 10^{\left[\frac{Mag_3db - Mag_1db}{20}\right]} \\ &= 10^{\left[\frac{23 - 23.4}{20}\right]} \\ &= 0.95 \end{aligned}$$

Notice that V_{ref} doesn't matter. The other coefficients are calculated in a similar manner.

[3]The spectrum harmonic amplitudes are shown here measured in db, which requires some calculation to determine the equivalent spectrum coefficients. The CGR-101 spectrum analysis amplitude can also be shown as a linear scale, which simplifies the calculations. However, the log (db) scale has a larger dynamic range, which makes small-amplitude harmonics more visible.

Knowing the values of the various spectrum coefficients, we can use the total RMS current and equation 7 to determine the magnitude of fundamental component. Modifying equation 7 to include the additional coefficients from the spectrum table we have:

$$I_T{}^2 = I_1{}^2 \left[1 + K_3{}^2 + K_5{}^2 + K_7{}^2 + K_9{}^2\right] \tag{9}$$

Rearrange to solve for the fundamental component of current:

$$I_1 = \sqrt{\frac{I_T{}^2}{1 + K_3{}^2 + K_5{}^2 + K_7{}^2 + K_9{}^2}} \tag{10}$$

Using the True RMS measurement feature of the CGR-101, shows a value of 111mV for the total current. The conversion factor of the IES-695 current meter is 100mV per amp. Then total RMS current is:

$$I_T = \frac{0.111}{0.1} = 1.11 \text{ amperes.}$$

Plug this value, with the spectrum coefficients, into equation 10 and we can determine the magnitude of the fundamental component of current I_1:

$$\begin{aligned} I_1 &= \sqrt{\frac{I_T{}^2}{1 + K_3{}^2 + K_5{}^2 + K_7{}^2 + K_9{}^2}} \\ &= \sqrt{\frac{1.11^2}{1 + 0.95^2 + 0.57^2 + 0.34^2 + 0.12^2}} \\ &= 0.722 \text{ amperes} \end{aligned}$$

The distortion power factor is given by the ratio of fundamental current to total current (squared). Substituting in equation 5 we have:

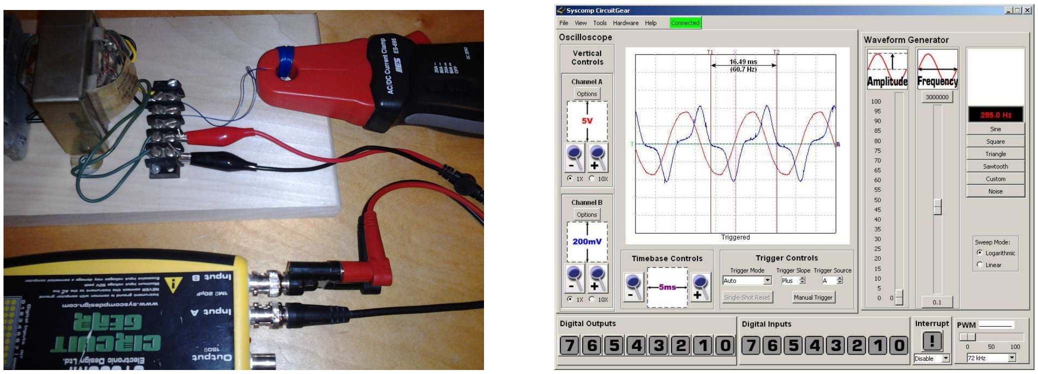
$$D = \frac{I_1}{I_T} = \frac{0.722}{1.11} = 0.65$$

This is a very value of distortion power factor, and indicates why some jurisdictions now require power factor correction for electronic power supplies[4].

Final Notes

A modern digital oscilloscope like the CGR-101, used in conjuction with a low-cost current probe, can measure both sinusoidal and non-sinusoidal power factor. Here are the features of the CGR-101 that supported these measurements:

- Dual channel digital oscilloscope display.
- Time and Ampitude cursors to measure waveforms.
- True RMS measurement.
- Waveform spectrum analysis.
- Screen capture to document results.



(a) Multiple Windings on Current Probe (b) Waveforms

Figure 6: Measuring Magnetizing Current

Appendix 1: Measuring the Transformer Magnetizing Current

We previously assumed that the magnetizing current of the adaptor-board transformer (figure 5) is negligible. Let's measure it to be certain.

The with a one-turn primary, the current probe is not sensitive enough to measure this current. But passing a number of turns N through the aperture of the clamp meter increases the sensitivity by the same factor.

We created a multi-turn primary by winding 30 turns of insulated wire-wrap wire onto the jaw of the current probe, as shown in figure 6(a). The current waveform is now large enough to measure easily, as shown in the screen-shot of figure 6(b). The vertical time cursors define one complete cycle for the measurement of RMS voltage from the current probe - according to the CGR-101 Auto Measurements panel, the value is 213mV. Invoking the volt-amp conversion factor of the current probe, and compensating for the number of turns on the primary, the transformer magnetizing current is:

$$I_{magnetizing} = \frac{0.213}{(0.1)(30)} = 0.017 \text{ amps (rms)}$$

This is the value of current that flows in the primary of the transformer when the secondary is open-circuited.

Notice that the current waveform is quite distorted. Evidently the line voltage is pushing the transformer operation into the extremes of the core BH curve.

Appendix 2: The Current Transformer

A *current transformer* is commonly used to measure current in an AC power system. It consists of a magnetic core, such as a toroid (doughnut), through which the current-carrying conductor passes. This is the one-turn primary winding of the transformer. The core is wound with a number N turns of insulated wire. This is the N turn secondary of the transformer.

This would be a step-up transformer for voltage (if the secondary was open circuited) and a step-down transformer for current.

A transformer reflects its secondary load resistance into the primary, by a factor of $1/N^2$. The current transformer should not reflect any significant resistance into the primary (because that would affect the current in the circuit) so the ideal secondary resistance is zero, a short circuit.

In industrial power applications, the secondary load is typically an ammeter, which appears as a short circuit. The current transformer steps down the current by its turns ratio N, so something like a 5 amp ammeter can be used to measure a much larger primary current.

[4] Those readers familiar with audio measurements will recognize the similarity of equation 10 to the equation for total harmonic distortion of an audio waveform.

In some cases, a small resistor called *the burden*, is attached to the secondary. Then the current transformer creates a voltage across this resistance which is proportional to the primary current. For the split-core transformer mentioned earlier [7], a 10 ohm burden resistor creates an output of 150mV for a primary current of 30 amps.

Notice that a secondary winding of a current transformer must *never* be open circuited when there is primary current. If it is open-circuited, the secondary voltage becomes very large. That may generate an arc with the possible breakdown of the winding insulation[5]

More information on the design of current transformers is in reference [3].

Acknowledgement

Special thanks to Harold Anderson who caught an error in a draft version of this paper and suggested a multiple-turn primary to increase the sensitivity of a current probe measurement. Harold also points out that the 5th and 7th harmonics in a current waveform have been known to cause resonance and consequent damage in power factor correction capacitors. This is especially a problem in installations with three-phase high current rectifier loads.

References

[1] Toronto Hydro, Power Factor Penalty
`http://www.gescanontario.com/wp-content/uploads/2012/07/An_Introduction_to_Power_Factor.pdf`

[2] *Syscomp Electronic Design Limited*
`www.syscompdesign.com`

[3] *Current Transformers: Part 1.*
David Knight, Ottery St Mary, Devon, UK.
`http://www.g3ynh.info/zdocs/bridges/Xformers/part_2.html`
This reference explains current transformers for use at radio frequencies, but the princibles are easily extrapolated to power system current transformers.

[4] *All About Circuits: Practical power factor correction*
`http://www.allaboutcircuits.com/vol_2/chpt_11/4.html`
The theory of power factor correction.

[5] Power Factor
`http://en.wikipedia.org/wiki/Power_factor`
A useful overview of power factor correction.

[6] IES 695 Current Meter (Clamp Meter)
`http://www.aeswave.com/Low-Current-Probe-p8987.html`).
Most clamp meters do not have an output that can display on an oscilloscope. Some clamp meters are AC only. The ES-695 is attractive because it is capable of measuring DC and AC current, it has a waveform output connection and it's reasonably priced. The frequency response is important in order that a complex waveform, which contains harmonics of the fundamental, be displayed accurately.the frequency response is 20kHz, which is adequate for AC line measurements. (It would not be adequate for switching power supply current measurements, where the switching frequency can be several MHz.)

[7] Split Core Current Sensor
`http://dlnmh9ip6v2uc.cloudfront.net/datasheets/Sensors/Current/ECS1030-L72-SPEC.pdf`

[8] Sparkfun Electronics
`https://www.sparkfun.com/products/11005`

[5]As part of this project the author inadvertently connected a 117:9 volt step-down transformer backwards, with the secondary attached to the AC line. The 'secondary' voltage would then have been around 1.5Kv. When power was applied the transformer made a sizzling noise and then flashed over in a puff of flame and smoke as the insulation broke down, destroying the transformer. Similar results could be expected on open-circuiting the secondary winding of a current transformer.