Hands-On Learning

Capacitor: Time Domain

Peter D. Hiscocks
Syscomp Electronic Design Limited
Email: phiscock@ee.ryerson.ca

October 15, 2014

Contents

1 Introduction

In this series of exercises we examine the electrical functions of the *capacitor*.

The capacitor is a device for storage of electronic charge (electrons). A battery is used for long-term storage of electronic charge. A capacitor is usually used for short-term storage. Furthermore, storing charge in a battery is inherently much less efficient than storing charge in a capacitor. Most of the charge into a capacitor is available on discharge, even at a very high discharge rate.

Capacitors are used as storage devices in power supplies, for filtering of alternating currents in power supplies and communications circuits, and in a wide variety of timing and waveform generation circuits.

2 Charging and Discharging by Constant Current

2.1 Theory: Charging by Constant Current

The fundamental equation describing capacitor operation:

$$Q_c = C\,V_c \tag{1}$$

where Q_c is the stored charge in coulombs, C is the capacitance in Farads, and V_c is the voltage between the terminals of the capacitor.

Differentiate each side of equation 1, and we have:

$$\frac{dQ_c}{dt} = C\frac{dV_c}{dt} \quad (2)$$

(Capacitance is a constant so it's not part of the differentiation.)

Now, dQ_c/dt is the current I_c into or out of the capacitor, so we can rewrite equation 2:

$$I_c = C\frac{dV_c}{dt} \quad (3)$$

This is a very useful form of the equation. It shows that there is a fundamental relationship between the current into or out of the capacitor, and the rate of change of the capacitor voltage. If the current is constant, then we could write:

$$I_c = C\frac{\Delta V}{\Delta t} \quad (4)$$

That is, the voltage across the capacitor increases at a constant rate. On an oscilloscope display, $\frac{\Delta V}{\Delta t}$ would be a linear ramp voltage. This relationship is useful in all kinds of applications – converting voltage to time, for example.

2.2 Exercise: Charging by Constant Current

We will illustrate this linear ramp effect by charging with a constant current source, which can be implemented by a junction field effect transistor (JFET).

The JFET has three-terminals: *Drain*, *Source* and *Gate*. When the source and gate are connected together, and a voltage applied between drain and source, the JFET conducts a current, technically known as I_{DSS}, the *drain-source saturation current*. Provided that the applied voltage is larger than some minimum (the *pinch off voltage*, typically a volt or two), this current is independent of the applied voltage. I_{DSS} varies from unit to unit (2 to 9 milliamps, for example), but for any given JFET it is a constant.

2.2.1 Step 1: Calibration

Before we begin measurements, it's a good idea to check the vertical calibration of the oscilloscope, so that it's not lying to you with an incorrect measurement.

Measure your voltage source (the 9 volt battery or lab power supply) with a digital voltmeter. Then connect this voltage source to the Channel A input of the oscilloscope. On the screen of the oscilloscope you should see the trace move vertically by approximately the correct number of divisions. For example, with a 9 volt source and the scope set to 2V per division vertical, the trace should move 4.5 divisions.

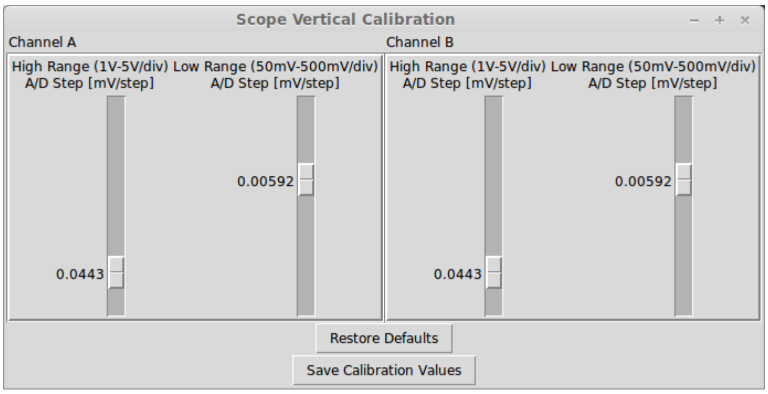


Figure 1: Vertical Calibration Panel

Now select the oscilloscope menu item `Tools -> Calibrate Scope Vertical Scale`. The control panel of figure 1 appears. Adjust the Channel A High Range slider until the vertical deflection of the Channel A trace is exactly correct. (It helps to enable the Channel A Cursors: menu item `View -> Channel A Cursors`.)

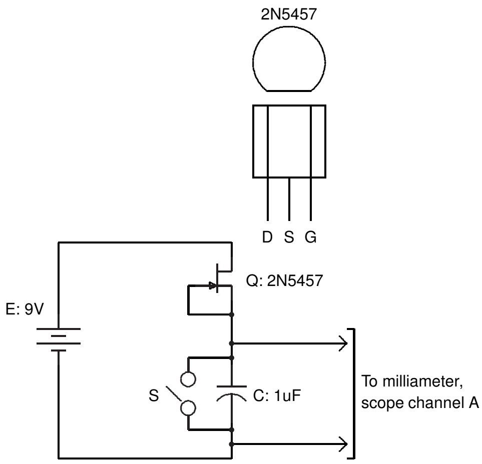
While you're at it, it's probably a good idea to calibrate Channel B as well, using a similar procedure.

2.2.2 Step 2. Wire the Test Circuit

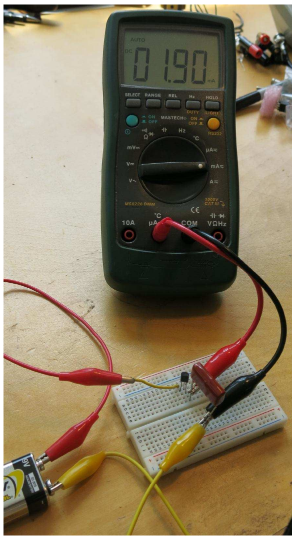
The ramp circuit is shown in figure 2(a).

The 2N5457 JFET was used in this circuit, but many other N-Channel JFETs will work just as well. Other values of capacitance will also work although it may be necessary to alter the oscilloscope timebase control to view the ramp. The switch can be a jumper wire or alligator clip wire.

Connect up the circuit. (Figure 2(b) may help in showing how to make the connections.)



(a) Ramp Circuit



(b) Measuring Current

Figure 2: Ramp Circuit Measurement

2.2.3 Step 3: Measure the Constant Current

Connect a milliammeter across the capacitor. The milliameter appears as (approximately) a short circuit, so the JFET current flows through the meter. Note the value of the JFET current. This will become the capacitor charging current I_c.

Current:

2.2.4 Step 4: Measure the Voltage Ramp

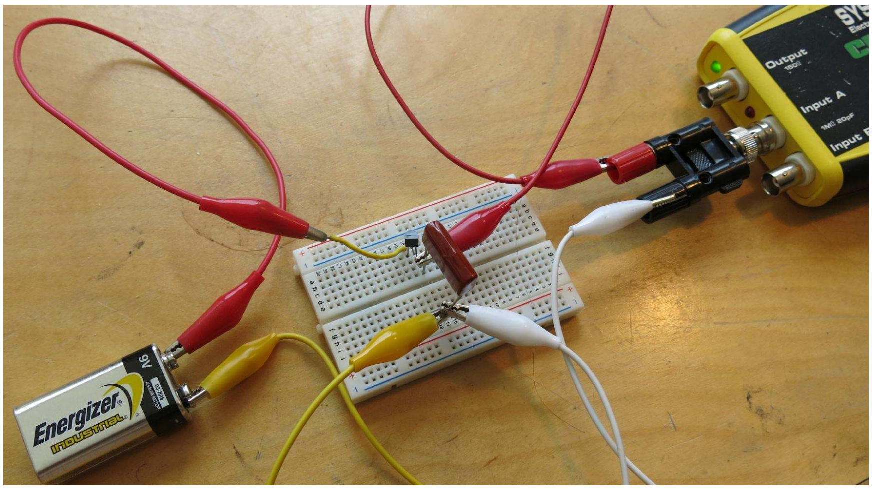
Remove the milliameter. Connect the oscilloscope Channel A to the terminals of the capacitor, figure 3(a).

Place a short circuit across the capacitor. (This is represented by switch S in the schematic of figure 2(a)). Check that the scope trace is at zero volts.

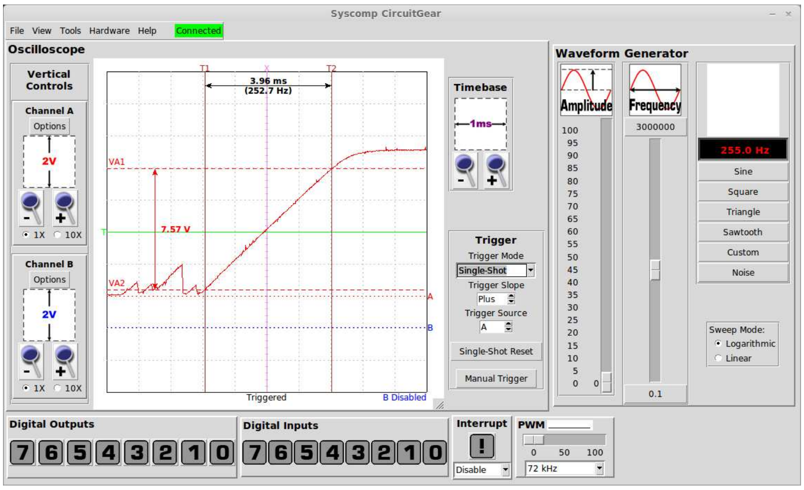
Remove the short circuit. The trace voltage should be around 9 volts.

Ensure that the trigger cursor is set half way between these two values, or around 4.5 volts. Connect the short circuit across the capacitor. Set the scope `Trigger Mode` to `Single Shot`. Click on `Single Shot Reset`. Remove the short circuit. The scope should capture a linear ramp similar to 3(b).

Using the time and voltage cursors, measure the voltage rate of change $\Delta V_c/\Delta T$ of the ramp.



(a) Scope Connection



(b) Ramp Waveform

Figure 3: Ramp Circuit Measurement Step 4

$\Delta V_c/\Delta T$:

2.2.5 Step 5: Calculate the Capacitance

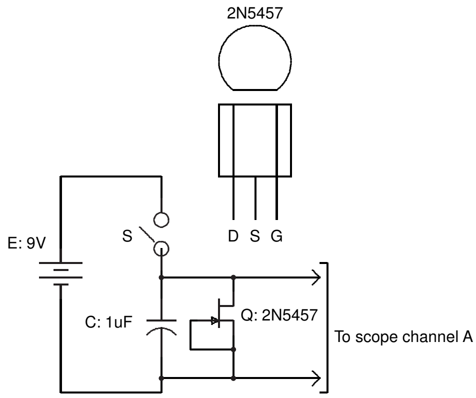
Use equation 4 and your measured values to calculate the value of the capacitor.

Capacitance:

Compare your measured value from the ramp calculation with the nominal value of the capacitance. (optional). If you have a capacitance measuring instrument, measure the capacitance of the capacitor.

Capacitance:

(a) Measuring the Capacitance



(b) Discharge Circuit

Figure 4:

2.3 Discharging by Constant Current

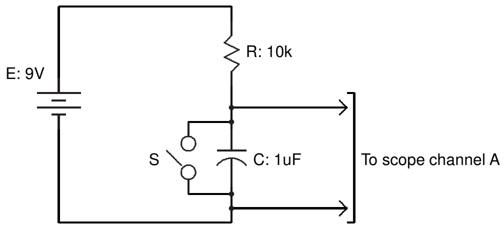
The capacitor may also be discharged by a constant current. Figure 4(b) shows the schematic diagram.

With the switch closed, the capacitor charges to the supply voltage. A constant current flows through the JFET. When the switch opens, this constant current discharges the capacitor.

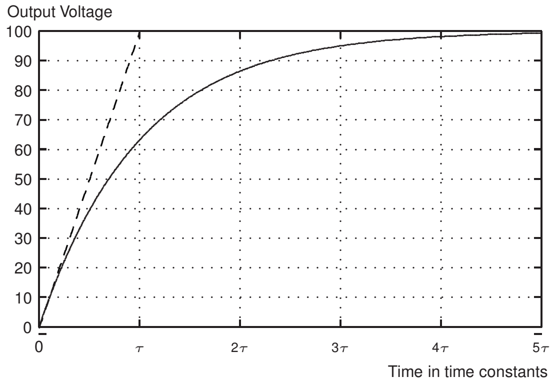
Using figure 3 as a guide, connect the oscilloscope Channel A across the capacitor.

Put the oscilloscope Trigger Mode to `Auto`. Verify that the capacitor voltage is equal to the supply voltage. Disconnect the supply voltage. Verify that the capacitor voltage drops to zero. Make sure the trigger level (T Cursor) is half way between the supply voltage and zero volts.

Reconnect the power supply to the capacitor and parallel JFET. Put the oscilloscope Trigger Mode to `Single Shot`. Set the trigger slope to `minus`. Make sure that single shot is `Reset`. Open the circuit to the power supply. The scope should capture a negative-going ramp voltage. Measure the slope and compare with the value of the positive-going slope measured earlier. Capture a screen shot of the oscilloscope trace and include with your results.



(a) Circuit



(b) Charging Curve

Figure 5: Resistor Charging Circuit

3 Charging and Discharging through a Resistor

3.1 Theory: Charging through a Resistor

Now consider the operation of the circuit when the constant-current device is replaced by a resistor, as shown in figure 5(a).

- We start with the switch closed. The voltage across the capacitor is zero.
- We open the switch. At that instant the capacitor voltage is zero. The voltage across the resistor is equal to the supply voltage E.
- Consequently by Ohm's law the current through the resistor is

$$i_c = \frac{E}{R}$$

- We have labelled this current i_c since it all flows into the capacitor. As current flows into the capacitor, its voltage rises.
- Consider that the capacitor charges for a very small time interval dt. Re-arranging equation 3, we have the change in voltage across the capacitor as:

$$dV_c = \frac{i_c}{C}\, dt \tag{5}$$

  In words, the voltage across the capacitor increases by an amount proportional to the charging current.
- After this time interval dt, the capacitor voltage has a small value dV_c. Kirchoff's voltage law implies that the voltage across the capacitor and across the resistor must add to equal the supply voltage. In other words, if the capacitor voltage increases, there is less voltage across the resistor.
- Because there is less voltage across the resistor:
  - The current through the resistor decreases.
  - The rate of change of voltage increase decreases, ie, the capacitor charges less quickly.

When the capacitor is charged by a constant current source, the voltage ramps up in a linear fashion. When the capacitor is charged through a resistor, the charging curve is not linear – it flattens out with time, as shown in figure 5(b). In fact, in theory, the capacitor is never totally completely charged[1] The equation describing the capacitor voltage may be shown [1] to be:

$$v_c(t) = E\left(1 - \epsilon^{-\dfrac{t}{RC}}\right) \quad (6)$$

The quantity RC is the so-called *time constant* of the circuit, given the symbol τ. A larger value of time constant causes a slower charging rate. Equation 6 is plotted in figure 5(b). This graph is completely general because (a) the scale of the vertical axis is in *percent charged*, that is, $v_c(t)/E$ as a percent, and (b) the scale of the time axis is in time constants.

Points of interest on this curve:

- At time $t = 0$, $v_c = 0$. The capacitor voltage starts at zero.
- At time $t = \infty$, $v_c = E$. The capacitor voltage ends at the supply voltage.
- At time $t = \tau$, $v_c = 0.63E$. The capacitor is 63% charged after one time constant, in seconds.
- The capacitor is very nearly completely charged after 5 time constants have elapsed.

Practice Calculations, Resistor Charging

These quentions refer to the circuit of figure 5(a).

What is the value of the time constant τ, in seconds?

Time Constant τ:

If the switch is closed and then opened at time $t = 0$, what is the voltage on the capacitor after 15 milliseconds?

v_c at $t = 15$msec:

The capacitor voltage is initially zero. What is the duration of the charging interval, in seconds, for the capacitor voltage to reach 4.5 volts?

t for $v_c = 4.5$ volts:

3.2 Exercise: Charging through a Resistor

Wire up the resistor charging circuit of figure 5(a).

Set up the oscilloscope as previously for the constant current ramp measurement:

Place a short circuit across the capacitor. (This is represented by switch S in the schematic of figure 5(a)). Check that the scope trace is at zero volts.

Remove the short circuit. The trace voltage should be around 9 volts.

[1] In practice, the charging current becomes equal to a very small *leakage current* through the capacitor or surrounding circuit, and there is no further change in voltage across the capacitor.

Ensure that the trigger cursor is set half way between these two values, or around 4.5 volts. Connect the short circuit across the capacitor. Set the scope `Trigger Mode` to `Single Shot`. Click on `Single Shot Reset`. Remove the short circuit. The scope should capture an exponential rising curve similar to 5(b).

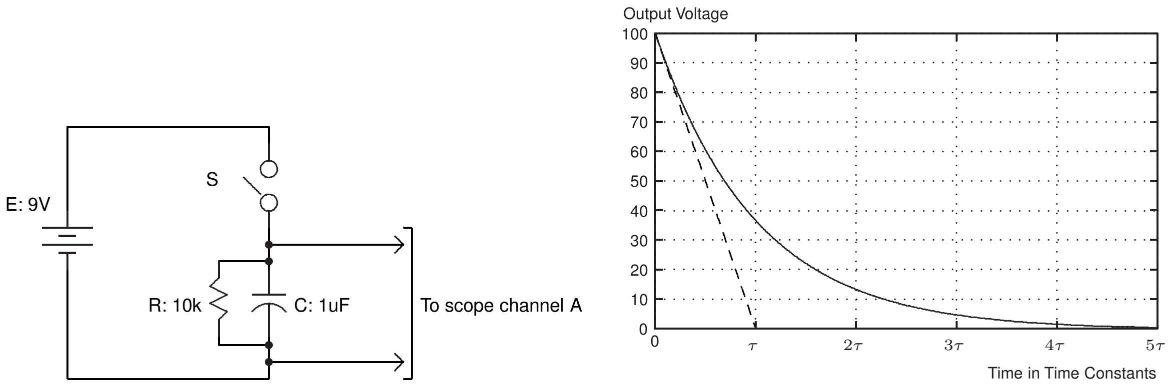
If the curve is too compressed, increase the *time per division* setting on the oscilloscope. If the curve is too expanded, decrease the setting.

Obtain a screen capture of the oscilloscope display, and retain for your report.

From the oscilloscope trace, determine the time constant of the circuit. Hint: Recall that the charging curve is equal to 63% of its final value at a time equal to one time constant.

τ:

3.3 Theory: Discharging Through a Resistor



(a) Circuit (b) Discharging Curve

Figure 6: Resistor Discharging Circuit

A similar line of reasoning applies to the discharge of a capacitor through a resistor. As the capacitor voltage decreases, the rate of discharge decreases, and the voltage-time curve flattens out. Figure 6(a) shows a test circuit. Figure 6(b) shows the generalized discharge curve.

The equation for a discharging capacitor is:

$$v_c(t) = E\,\epsilon^{-\dfrac{t}{RC}} \tag{7}$$

For example, at $t = RC$ (one time constant), the capacitor voltage is 0.36 E, that is, 36% discharged.

3.4 Exercise: Discharging through a Resistor

Wire up the resistor charging circuit of figure 6(a).

Set up the oscilloscope as previously for the constant current ramp measurement:

Close the switch S (or use a jumper wire to complete that connection). Check that the scope trace is indicating the supply voltage, around 9 volts.

Open the switch (or disconnect the jumper lead). The trace voltage should be around 0 volts.

Ensure that the trigger cursor is set half way between these two values, or around 4.5 volts. Close the switch. Set the scope `Trigger Mode` to `Single Shot`. Ensure that the trigger slope is set to *minus*, ie, negative slope. The timebase setting you used for the charging circuit should be correct for this circuit. As before, if the curve is too compressed increase the *time per division* setting on the oscilloscope. If the curve is too expanded, decrease the setting.

Click on `Single Shot Reset`. Open the switch. The scope should capture an exponential falling curve similar to 6(b).

Obtain a screen capture of the oscilloscope display, and retain for your report.

From the oscilloscope trace, determine the time constant of the circuit. Hint: Recall that the charging curve is equal to 36% of its final value at a time equal to one time constant.

τ:

Compare with the value of time constant that you obtained for the charging circuit.

References

[1] *A Laplace Transform Cookbook*
Peter Hiscocks
`http://www.syscompdesign.com/assets/Images/AppNotes/laplace-cookbook.pdf`